Mathematical Model and Algorithm for Optimizing Configuration and Operation of Energy System

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This paper proposes an optimization method to find an optimal configuration and operation of energy system for facilities including factories and plants that consume energy in large quantities from the viewpoint of energy cost reduction with combining batteries, photovoltaic generation systems, and private power generators. It is not easy to optimize configuration and operation of energy systems since it is necessary to consider synergistic and complementary effects between pieces of energy equipment as well as the performance and the cost of the individual equipment. In addition to this, recent lively utilization of renewable energies increases options of configuration and it causes the problem to be more complicated. For this issue, we propose a general-purpose mathematical model and algorithm for optimization of energy system configuration and operation that considers implementation and provision as an optimization system that can be used for various projects.

1. Introduction

At many factories, plants, and other facilities where a large amount of energy is consumed, energy systems consisting mainly of generators, batteries, and photovoltaic generation systems are specially configured and operated in order to reduce energy cost and environmental burden. When examining the configuration and operation of energy systems, optimization is not an easy matter, since it is necessary to discuss not only the performance and cost of each piece of energy equipment constituting a candidate for installation, but also the combination of equipment that enables full utilization of each item's advantages and compensates for its disadvantages. In addition to this, recent lively development and utilization of renewable energies[1] make it more complicated to determine the configuration and operation of optimal energy systems.

A very large amount of research has been performed on optimizing the configuration and operation of energy systems, details of which can be found in survey articles. For example, Reference (2) focuses on objective functions, constraints, and formulation, while Reference (3) focuses on the modeling of energy equipment. Many articles discuss operation optimization, with some focusing on electricity as energy and batteries as equipment[4], [5], while others cover various types of energy and equipment[6]. With regard to configuration optimization that considers how individual pieces of equipment should be operated, some optimization methods have been proposed that achieve a good balance between solvability and model accuracy[7], [8].

Not all energy system engineers are experts of mathematical optimization, actually, and it is therefore unrealistic for them to conduct modeling and algorithm implementation for each project on their own. It is desirable that a series of optimization technologies be provided as an optimization system which only requires the efficiency, cost information, and other values for each piece of energy equipment. There are not many studies which focus on this.

With respect to this issue, this paper proposes a versatile mathematical model and an algorithm for solving the mathematical model. They can be implemented and provided as an optimization system for optimizing the configuration and operation of energy systems in various projects. Using the optimization system above eliminates the need to develop a model and/or an algorithm for each project, so that more efficient investigation of energy system configuration and operation optimization may be expected. In addition, even when new renewable energies and energy equipment that uses them emerge, it will be possible to make immediate proposals regarding how these can be effectively combined.

In this paper, the electricity, heat, gas, fuel, CO_2, and other substances and energy that are input into or output from energy equipment are collectively referred to as "resources." A particular feature of the model proposed in this paper is that the characteristics of various types of energy equipment are represented as numerical parameters related to the input and output characteristics of resources. This makes it possible to easily handle new equipment and resource types by changing these parameter values. It is also a feature of the proposed model that it does not require technical knowledge of mathematical optimization to use the system due to the clarity of definitions of these parameters means.

The proposed model is formulated as a mixed integer programming problem. Although small problem instances can be solved by a mathematical programming solver, problems may be relatively large due to the nature of a versatile model. We propose an approximate algorithm for large problem instances based on the Benders decomposition method[9] exploiting the characteristics of the problem structure.

This paper is organized as follows. **Chapter 2** gives an overview of the energy system configuration and operation optimization problem discussed in this paper, while **Chapter 3** details the technical requirements for the optimization technology considering use-case, accuracy, and expandability, and then presents formulation of the optimization problem. **Chapter 4** discusses the structure of the optimization problem formulated in **Chapter 3** and presents an efficient optimization algorithm based on the Benders decomposition method. **Chapter 5** describes the optimization system based on the proposed model and algorithm, and presents an example calculation using the system. Finally, **Chapter 6** gives a summary of this paper and describes future challenges.

2. Energy system configuration and operation optimization problem

This chapter discusses how to configure and operate an energy system that optimizes cost with respect to energy demand (e.g., electricity, heat) at a factory, plant, or similar facility. **Figure 1** shows an overview of this optimization problem. It is assumed that we can refer information related to the candidate equipment and energy. The information about the equipment includes rated output and capacity ranges, efficiency, construction cost, and maintenance cost, and that about the energy includes the demand pattern for a typical day and its annual growth rate, and cost. Based on the given information shown above, the optimization problem discussed in this paper is solved by determining the configuration (necessity of introducing each piece of candidate equipment, as well as output and capacity) of an energy system that minimizes the sum of initial cost and running cost for a specified number of years, giving consideration to operation on a typical day.

3. Modeling

3.1 Definition of requirements

The optimization discussed in the paper is used to roughly determine the type, size (e.g., rated output, capacity), and cost of energy equipment optimal for expected energy demand in the early design stage of an energy system, and the following four requirements are adopted:

(1) The optimization should have high expandability to cover various types of equipment and energy (e.g., electricity, heat, hydrogen), and should not require modification of a program when new equipment or energy types are added.
(2) The optimization should take account of various constraints (e.g., contract demand, CO_2 emissions, equipment operation hours).
(3) High-speed calculation should be possible (a few seconds to approximately one minute).
(4) The obtained solution should be capable of being explained.

In consideration of these requirements, this paper formulates the problem as a mixed integer programming problem. This provides high expandability and enables high-speed optimization calculation through designing algorithm that exploits the problem structure. With regard to the above-mentioned requirement (4), the key points of the solution, such as sensitivity to uncertainty in demand and critical constraints, and their contribution to cost, can be obtained by defining and solving appropriate dual problems. The detailed formulation as the mixed integer programming problem is described in **Section 3.3**.

3.2 Basic concept of calculation model

3.2.1 Resource balance

In consideration of expandability, which is one of the requirements given in **Section 3.1**, this paper refers to the materials and energy that are input into and output from energy equipment as “resources,” and classifies energy

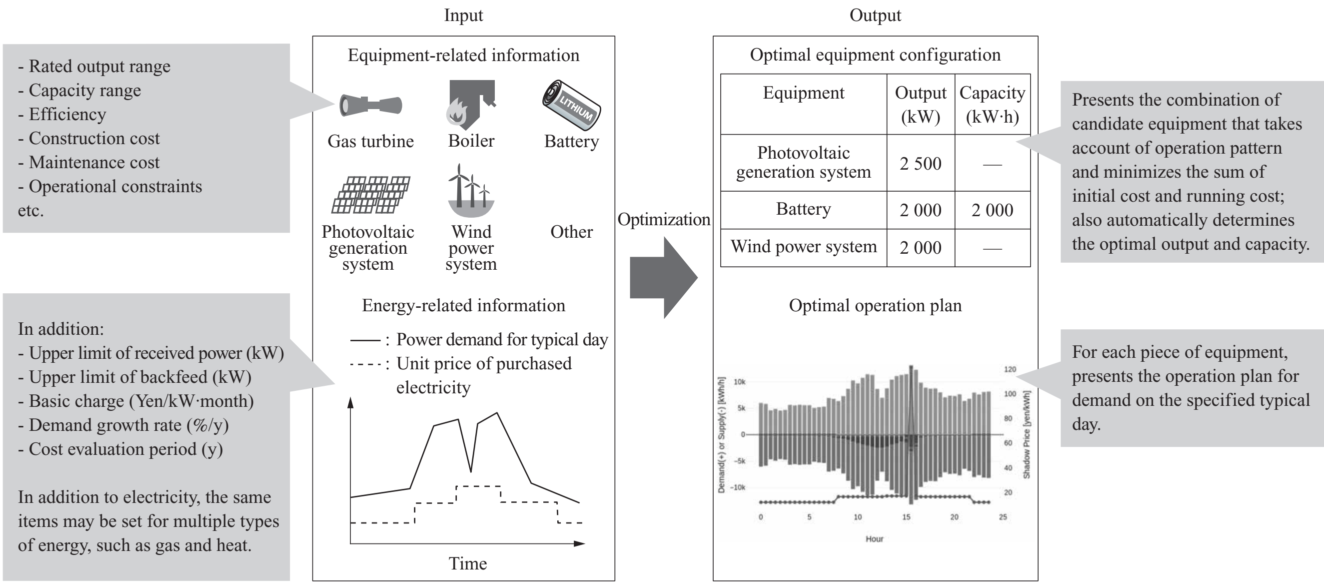


Fig. 1 Conceptual drawing of energy system configuration and operation optimization problem

equipment into the following three types:

(1) Converter: Equipment that converts one or plural resource into others (e.g., gas turbine, water electrolyzer)
(2) Storage: Equipment that stores a resource (e.g., battery)
(3) Renewable energy: Equipment that generates a resource without any input (e.g., photovoltaic generation system)

This paper employs a model that represents the characteristics of each piece of equipment by numerical parameters. For example, a gas engine is regarded as equipment that consumes a resource (gas) and generates three resources (electricity, heat, and CO_2), and its characteristics are represented by numerical data that gives the amount of gas consumption and amounts of electricity, heat, and CO_2 generations when it is operated at unit output for a unit period of time. **Figure 2** shows a conceptual drawing of specific equipment modeling taking a gas engine as an example. In **Fig. 2**, the amount of gas consumed c_{gas}, and amounts of electricity, heat, and CO_2 generated $g_{electricity}$, g_{heat}, and g_{CO_2} define the characteristics of the gas engine considered as a piece of energy equipment. An advantage of adopting such a model is that it can easily be used for new equipment and resources simply by changing the numerical parameters.

When an energy system is operated, there is a balance equation that holds for each resource at each time.

(Input from the outside of the system) + (Generated by equipment)
= (Consumed by equipment) + (Demand) + (Output to the outside of the system) ··································(1)

Expression (1) is given as an equality constraint in the optimization problem. In the proposed model, the operation cost of the energy system is determined by the input from the outside of the system and output to the outside of the system. For example, the input from the outside of the system includes the cost of purchasing electricity from an electric power company and the cost of purchasing gas from a gas company, and the output to the outside of the system includes the cost of discharging CO_2 outside the factory. If an input or output exceeds a specified value, a penalty cost may be incurred. An additional cost may be incurred that depends on the maximum input or output value during operation. To simplify discussion, these are not taken into consideration in the mathematical model discussed in the following, but can easily be reflected through simple expansion of the model, and the proposed algorithm can also be applied without modifications.

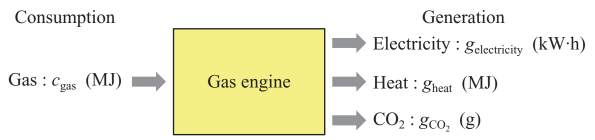


Fig. 2 Conceptual drawing of modeling of energy equipment

3.2.2 Cost

In the optimization, the objective function to be minimized is defined by sum of the initial cost of the introduced energy equipment and running cost for an appropriate number of years. The initial cost includes the construction cost of the equipment. The running cost is further classified into maintenance cost and operation cost. The maintenance cost is required regardless of the operation status of the equipment as long as the equipment is owned, and it includes repair cost, fixed asset tax, and labor cost. The operation cost changes depending on the operation status, and it is determined by the input from the outside of the energy system and output to the outside of the system. The operation cost includes electricity purchase cost and gas purchase cost.

Figure 3 shows a conceptual drawing of cost breakdown that summarizes the above discussion. As shown in **Fig. 3**, the initial cost and maintenance cost depend only on equipment configuration, and the operation cost depends on operation of introduced equipment. In **Chapter 4**, this cost classification and dependence relationships are used to design an efficient optimization algorithm.

3.3 Formulation

Based on the concepts described in **Section 3.2**, the optimization problem is formulated as a mixed integer programming problem as follows:

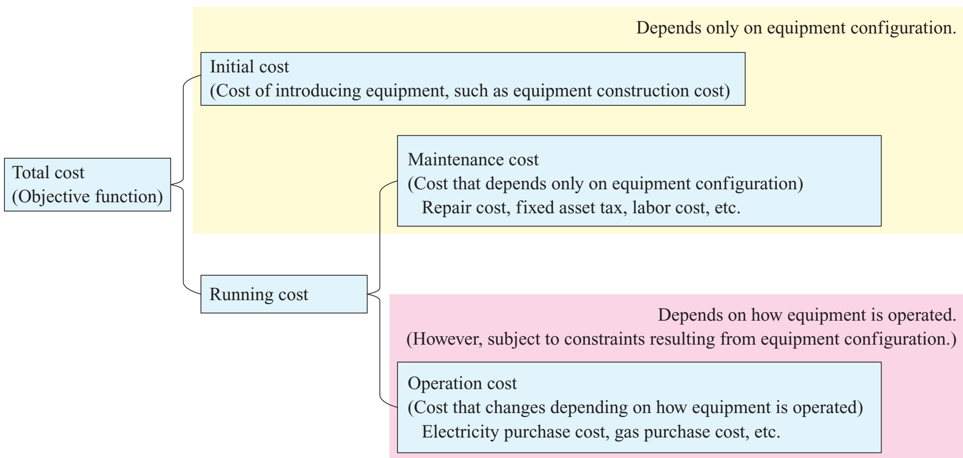


Fig. 3 Conceptual drawing of cost structure

$$\text{(P)}: \underset{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{p}, \boldsymbol{p}^+, \boldsymbol{p}^-, \boldsymbol{q}_0, \boldsymbol{h}, \boldsymbol{s}^+, \boldsymbol{s}^-}{\text{minimize}} \ f^{\mathrm{I}}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) + \sum_{k \in \mathcal{K}} f_k^{\mathrm{R}}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{s}_k^+, \boldsymbol{s}_k^-) \quad \cdots\cdots (2)$$

subject to

$$x_i^{\min} z_i \le x_i \le x_i^{\max} z_i \left(i \in \mathcal{E}\right), \quad \cdots\cdots (3)$$

$$y_i^{\min} z_i \le y_i \le y_i^{\max} z_i \left(i \in S\right), \quad \cdots\cdots (4)$$

$$r_{il}^{\min p} x_i - \left(1 - h_{ikl}\right) x_i^{\max} \le p_{ikl} \le r_{il}^{\max p} x_i \quad \left(i \in C, k \in \mathcal{K}, l \in \mathcal{L}\right), \quad \cdots\cdots (5)$$

$$0 \le p_{ikl} \le r_{il}^{\max p} x_i^{\max} h_{ikl} \quad \left(i \in C, k \in \mathcal{K}, l \in \mathcal{L}\right), \quad \cdots\cdots (6)$$

$$0 \le p_{ikl}^+ \le r_{il}^{\max p} x_i \quad \left(i \in S, k \in \mathcal{K}, l \in \mathcal{L}\right), \quad \cdots\cdots (7)$$

$$0 \le p_{ikl}^- \le r_{il}^{\max p} x_i \quad \left(i \in S, k \in \mathcal{K}, l \in \mathcal{L}\right), \quad \cdots\cdots (8)$$

$$p_{ikl}^+ \le r_{il}^{\max p} x_i^{\max} h_{ikl} \quad \left(i \in S, k \in \mathcal{K}, l \in \mathcal{L}\right), \quad \cdots\cdots (9)$$

$$p_{ikl}^- \le r_{il}^{\max p} x_i^{\max} \left(1 - h_{ikl}\right) \quad \left(i \in S, k \in \mathcal{K}, l \in \mathcal{L}\right), \quad \cdots\cdots (10)$$

$$r_{il}^{\min q} y_i \le q_{ikl}\left(\boldsymbol{p}_{ik}^+, \boldsymbol{p}_{ik}^-, q_{ik0}\right) \le r_{il}^{\max q} y_i \quad \left(i \in S, k \in \mathcal{K}, l \in \mathcal{L}\right), \quad \cdots\cdots (11)$$

$$q_{ik|L-1|}(\boldsymbol{p}_{ik}^+, \boldsymbol{p}_{ik}^-, q_{ik0}) = q_{ik0} \quad \left(i \in S, k \in \mathcal{K}\right), \quad \cdots\cdots (12)$$

$$0 \le s_{nkl}^+ \le s_n^{+\max} \quad \left(n \in \mathcal{N}, k \in \mathcal{K}, l \in \mathcal{L}\right), \quad \cdots\cdots (13)$$

$$0 \le s_{nkl}^- \le s_n^{-\max} \quad \left(n \in \mathcal{N}, k \in \mathcal{K}, l \in \mathcal{L}\right), \quad \cdots\cdots (14)$$

$$\sum_{i \in C} g_{in} p_{ikl} + \sum_{i \in S} g_{in} p_{ikl}^- + \sum_{i \in \mathcal{R}} g_{in} \bar{r}_{il} x_i + s_{nkl}^- = \sum_{i \in C} c_{in} p_{ikl} + \sum_{i \in S} c_{in} p_{ikl}^+ + s_{nkl}^+ + d_{nkl} \quad \left(n \in \mathcal{N}, k \in \mathcal{K}, l \in \mathcal{L}\right), \quad \cdots\cdots (15)$$

$$z_i \in \{0, 1\} \left(i \in \mathcal{E}\right), \quad \cdots\cdots (16)$$

$$h_{ikl} \in \{0, 1\} \left(i \in C \cup S, k \in \mathcal{K}, l \in \mathcal{L}\right). \quad \cdots\cdots (17)$$

The meaning of each symbol is as shown in **Tables 1 to 5**. The symbol "u_n" under the "Unit" column in **Table 5** indicates a unit that depends on the type of resource. For example, "u_n" is "MJ" for gas and fuel, and "m^3" for water. Each of the dependent variables given in **Table 4** is calculated as follows:

$$f^{\mathrm{I}}\left(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\right) = \sum_{i \in \mathcal{E}} \left(\alpha_i^0 x_i + \gamma_i^0 z_i\right) + \sum_{i \in S} \beta_i^0 y_i, \quad \cdots\cdots (18)$$

$$f_k^{\mathrm{R}}\left(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{s}_k^+, \boldsymbol{s}_k^-\right) = f_k^{\mathrm{M}}\left(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\right) + f_k^{\mathrm{O}}\left(\boldsymbol{s}_k^+, \boldsymbol{s}_k^-\right) \quad \left(k \in \mathcal{K}\right), \quad \cdots\cdots (19)$$

$$f_k^{\mathrm{M}}\left(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\right) = \sum_{i \in \mathcal{E}} \left(\alpha_i^k x_i + \gamma_i^k z_i\right) + \sum_{i \in S} \beta_i^k z_i \quad \left(k \in \mathcal{K}\right), \quad \cdots\cdots (20)$$

$$f_k^{\mathrm{O}}\left(\boldsymbol{s}_k^+, \boldsymbol{s}_k^-\right) = D \Delta T \sum_{l \in \mathcal{L}} \sum_{n \in \mathcal{N}} \left(\phi_{nl}^+ s_{nkl}^+ + \phi_{nl}^- s_{nkl}^-\right) \quad \left(k \in \mathcal{K}\right), \quad \cdots\cdots (21)$$

Table 1 Definitions of Symbols (Sets)

Symbol	Description
$\mathcal{E}$	Set of equipment
C	Set of converters ($C \subseteq \mathcal{E}$)
S	Set of storages ($S \subseteq \mathcal{E}$)
$\mathcal{R}$	Set of renewable energies ($\mathcal{R} \subseteq \mathcal{E}$)
$\mathcal{K}$	Set of years ($\mathcal{K} = \{1, \ldots, \lvert\mathcal{K}\rvert\}$)
$\mathcal{L}$	Set of time steps ($\mathcal{L} = \{0, 1, \ldots, \lvert\mathcal{L}\rvert - 1\}$) (0 to 24 o'clock on day with typical demand pattern)
$\mathcal{N}$	Set of resources

Table 2 Definitions of Symbols (Constants)

Symbol	Unit	Description
D	d/y	Number of days in year (365 d/y)
ΔT	h	Duration of one time step

Table 3 Definitions of Symbols (Decision Variables)

Symbol	Unit	Description
x_i	kW	Output of equipment i
y_i	kW·h	Capacity of equipment i ($i \in S$)
z_i	—	1 when equipment i is introduced, and 0 when not introduced
p_{ikl}	kW	Operation output of equipment i at step l on typical day in k-th year ($i \in C$)
p_{ikl}^+	kW	Operation output of equipment i at step l on typical day in k-th year (charge side) ($i \in S$)
p_{ikl}^-	kW	Operation output of equipment i at step l on typical day in k-th year (discharge side) ($i \in S$)
q_{ik0}	kW·h	Initial remaining energy value of equipment i on typical day in k-th year ($i \in S$)
h_{ikl}	—	Operation status of equipment i at step l on typical day in k-th year (Running: 1, stopped: 0 ($i \in C$). charge: 1, discharge: 0 ($i \in S$))
s_{nkl}^+	kW	Speed of output of resource n to the outside of the system at step l on typical day in k-th year
s_{nkl}^-	kW	Speed of input of resource n from the outside of the system at step l on typical day in k-th year

Table 4 Definitions of Symbols (Dependent Variable)

Symbol	Unit	Description
f^{I}	Yen	Initial cost, including equipment investment
f_k^{R}	Yen	Running cost in k-th year
f_k^{M}	Yen	Maintenance cost in k-th year
f_k^{O}	Yen	Operation cost in k-th year
q_{ikl}	kW·h	Remaining energy of equipment i step l on typical day in k-th year ($i \in S$)

Table 5 Definitions of Symbols (Parameters)

Symbol	Unit	Description
$x_i^{\min}$	kW	Lower limit of rated output of equipment i
$x_i^{\max}$	kW	Upper limit of rated output of equipment i
$y_i^{\min}$	kW·h	Lower limit of capacity of equipment i ($i \in S$)
$y_i^{\max}$	kW·h	Upper limit of capacity of equipment i ($i \in S$)
$r_{il}^{\min p}$	—	Lower limit of operation output of equipment i at step l (ratio against rated output) ($i \in C$)
$r_{il}^{\max p}$	—	Upper limit of operation output of equipment i at step l (ratio against rated output) ($i \in C$)
$r_{il}^{\min q}$	—	Lower limit of remaining energy of equipment i at step l (ratio against capacity) ($i \in S$)
$r_{il}^{\max q}$	—	Upper limit of remaining energy of equipment i at step l (ratio against capacity) ($i \in S$)
$s_n^{+\max}$	u_n/h	Upper limit of speed of output of resource n to outside of system
$s_n^{-\max}$	u_n/h	Upper limit of speed of input of resource n from outside of system
g_{in}	u_n/kW·h	Amount of resource n generated when equipment i is operated at unit output for unit time
c_{in}	u_n/kW·h	Amount of resource n consumed when equipment i is operated at unit output for unit time
d_{nkl}	u_n/h	Demand for resource n at step l on typical day in k-th year
$\bar{r}_{il}$	—	Operation output of equipment i at step l (ratio against equipment output) ($i \in \mathcal{R}$)
α_i^0	Yen/kW	Equipment investment cost of equipment i (constant of proportionality against rated output)
β_i^0	Yen/kW·h	Equipment investment cost of equipment i (constant of proportionality against capacity) ($i \in S$)
γ_i^0	Yen	Equipment investment cost of equipment i (cost incurred regardless of output or capacity)
α_i^k	Yen/kW	Maintenance cost of equipment i in k-th year (constant of proportionality against rated output)
β_i^k	Yen/kW·h	Maintenance cost of equipment i in k-th year (constant of proportionality against capacity) ($i \in S$)
γ_i^k	Yen	Maintenance cost of equipment i in k-th year (cost incurred when introduced regardless of output or capacity)
ϕ_{nl}^+	Yen/u_n	Cost to output unit amount of resource n to outside of system at step l
ϕ_{nl}^-	Yen/u_n	Cost to input unit amount of resource n from outside of system at step l

$$q_{ikl}\left(\boldsymbol{p}_{ik}^+, \boldsymbol{p}_{ik}^-, \boldsymbol{q}_{ik0}\right) = q_{ik0} + \Delta T \sum_{m \in \mathcal{L}, m \leq l} \left(p_{ikm}^+ - p_{ikm}^-\right)$$

$$\left(i \in S,\ k \in \mathcal{K},\ l \in \mathcal{L}\right). \quad \cdots\cdots(22)$$

In Expressions (2) to (22) and hereinafter, a bold symbol from which some or all of the subscripts seen in **Tables 1 to 5** are omitted indicates a vector that contains all information about the omitted subscripts. The meanings of Expressions (2) to (17), which describe the objective function and constraints, are as follows:

Expression (2) : The objective function is the sum of the initial cost and running cost for $|\mathcal{K}|$ years.

Expression (3) : The rated output of equipment is determined between the specified upper and lower limits.

Expression (4) : The capacity of equipment is determined between the specified upper and lower limits.

Expressions (5), (6) : The operation output of each converter is 0 or is determined between the specified upper and lower limits.

Expressions (7), (8) : The output of each storage (charge side and discharge side) is determined between the specified upper and lower limits.

Expressions (9), (10) : No storage can store and discharge energy at the same time.

Expression (11) : Each storage is operated between the specified upper and lower limits for remaining energy.

Expression (12) : The remaining energy of each storage returns to the initial value after being operated for one day.

Expressions (13), (14) : The system external output (input) of each resource is determined between the specified upper and lower limits.

Expression (15) : A balance equation holds between the generation, input, consumption, output, and demand of each resource at each time of each year.

Expressions (16), (17) : The value of decision variables z_i and h_{ikl} must be 0 or 1.

The optimization problem (P) has been modeled without determining specific resources or equipment, and possesses scalability with respect to these. By implementing of this model as an optimization system in combination with the algorithm proposed in **Chapter 4**, optimization calculation can be performed simply by setting the parameters listed in **Table 5** according to the types and numbers of resources and equipment, rather than requiring different models and algorithms for different projects. In addition, each parameter can easily be interpreted physically, and optimization calculation can therefore be conducted by an engineer who is not an expert of mathematical optimization.

4. Optimization algorithm

The optimization problem (P) is written as a mixed integer programming problem, hence solution can be attempted using a general-purpose mathematical programming solver.

For a relatively small problem instance, optimization can be achieved with a mathematical programming solver. However, when the cost consideration period $|\mathcal{K}|$ is relatively long and fluctuations in demand depending on year are taken into account, calculation may take a long period of one or more days. Incidentally, the optimization discussed in this paper is used as a rough calculation, therefore accurate optimization is not necessarily required. In consideration of this point, this chapter proposes an algorithm based on the Benders decomposition method that solves the optimization problem (P) quickly and approximately. The proposed algorithm makes it possible to perform an approximate optimization for even a relatively large problem instance in only a few seconds to approximately one minute.

Examining the structure of the objective function for the optimization problem (P) shows that the function is the sum of terms that depend only on the equipment configuration ($\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z}$), (i.e., initial cost f^{I}, maintenance cost f_k^{M}), and terms that also depend on $(\boldsymbol{s}_k^+, \boldsymbol{s}_k^-)$ in operation plan (i.e., operation cost f_k^{O} $(k \in \mathcal{K})$. Note that the latter terms are independent in each year if equipment configuration $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ is fixed. Now, consider the following procedure:

(1) First, use some method to set the equipment configuration as $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{z}})$.

(2) For $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{z}})$, solve the operation plan optimization problem independently for each year (may be calculated in parallel).

(3) Feed back the information from the obtained solution into optimization of equipment configuration (go back to (1)).

The Benders decomposition method is known as an algorithm that employs this kind of problem decomposition procedure[9]. Benders decomposition method is applicable if the problem has following structural characteristics:

- The objective function and constraint functions of the problem can be partitioned into linear terms and other terms, which include nonlinear function terms and 0-1 variables terms.
- The linear part of the partitioned problem is solvable as a linear programming problem if the variables related to the other parts are fixed.

Using the structural characteristics above, Benders decomposition can solve the original problem in finite iterations with successively tightening its lower bound based on the dual optimal solutions of the linear programming problems. In the problem (P), however, when the equipment configuration is determined and the operation plan optimization is split, 0-1 variables remain in individual operation plan optimization problems, hence typical Benders decomposition cannot be applied. Therefore, in this paper, we discuss a method of obtaining the lower bound based on the continuous relaxation problem for the operation plan optimization problem. Although solutions obtained by the proposed algorithm have no guarantee of optimality for the original problem (P), they can be expected as good feasible solutions computed in a short time.

The following is the operation plan optimization problem in the k-th year with the equipment configuration fixed as $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{z}})$:

$$\left(\mathrm{P}_k(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{z}})\right): \underset{\boldsymbol{p}_k, \boldsymbol{p}_k^+, \boldsymbol{p}_k^-, \boldsymbol{q}_{k0}, \boldsymbol{h}_k, \boldsymbol{s}_k^+, \boldsymbol{s}_k^-}{\text{minimize}} \; f_k^{\mathrm{O}}\left(\boldsymbol{s}_k^+, \boldsymbol{s}_k^-\right) \quad \cdots\cdots\cdots(23)$$

subject to

$$r_{il}^{\min p}\bar{x}_i - \left(1 - h_{ikl}\right)x_i^{\max} \le p_{ikl} \le r_{il}^{\max p}\bar{x}_i$$
$$\left(i \in C,\, l \in \mathcal{L}\right), \quad \cdots\cdots(24)$$

$$0 \le p_{ikl} \le r_{il}^{\max p} x_i^{\max} h_{ikl} \quad \left(i \in C,\, l \in \mathcal{L}\right), \quad \cdots\cdots(25)$$

$$0 \le p_{ikl}^+ \le r_{il}^{\max p}\bar{x}_i \quad \left(i \in S,\, l \in \mathcal{L}\right), \quad \cdots\cdots(26)$$

$$0 \le p_{ikl}^- \le r_{il}^{\max p}\bar{x}_i \quad \left(i \in S,\, l \in \mathcal{L}\right), \quad \cdots\cdots(27)$$

$$p_{ikl}^+ \le r_{il}^{\max p} x_i^{\max} h_{ikl} \quad \left(i \in S,\, l \in \mathcal{L}\right), \quad \cdots\cdots(28)$$

$$p_{ikl}^- \le r_{il}^{\max p} x_i^{\max}\left(1 - h_{ikl}\right) \quad \left(i \in S,\, l \in \mathcal{L}\right), \quad \cdots\cdots(29)$$

$$r_{il}^{\min q}\bar{y}_i \le q_{ikl}\left(\boldsymbol{p}_{ik}^+, \boldsymbol{p}_{ik}^-, \boldsymbol{q}_{ik0}\right) \le r_{il}^{\max q}\bar{y}_i$$
$$\left(i \in S,\, l \in \mathcal{L}\right), \quad \cdots\cdots(30)$$

$$q_{ik|L-1|}\left(\boldsymbol{p}_{ik}^+, \boldsymbol{p}_{ik}^-, \boldsymbol{q}_{ik0}\right) = q_{ik0} \quad \left(i \in S\right), \quad \cdots\cdots(31)$$

$$0 \le s_{nkl}^+ \le s_n^{+\max} \quad \left(n \in \mathcal{N},\, k \in \mathcal{K},\, l \in \mathcal{L}\right), \quad \cdots\cdots(32)$$

$$0 \le s_{nkl}^- \le s_n^{-\max} \quad \left(n \in \mathcal{N},\, k \in \mathcal{K},\, l \in \mathcal{L}\right), \quad \cdots\cdots(33)$$

$$\sum_{i \in C} g_{in} p_{ikl} + \sum_{i \in S} g_{in} p_{ikm}^- + \sum_{i \in \mathcal{R}} g_{in}\bar{r}_{il}\bar{x}_i + s_{nkl}^-$$
$$= \sum_{i \in C} c_{in} p_{ikl} + \sum_{i \in S} c_{in} p_{ikm}^+ + s_{nkl}^+ + d_{nkl}$$
$$\left(n \in \mathcal{N},\, k \in \mathcal{K},\, l \in \mathcal{L}\right), \quad \cdots\cdots(34)$$

$$z_i \in \{0, 1\} \left(i \in \mathcal{E}\right). \quad \cdots\cdots(35)$$

Since $\left(\mathrm{P}_k(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{z}})\right)$ is a small-scale optimization problem concerning only one year (typical days), it can be solved in a relatively short time using an appropriate mathematical programming solver. The following discussion assumes that the existence of an optimal solution of $\left(\mathrm{P}_k(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{z}})\right)$ is guaranteed for any equipment configuration $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{z}})$.

Consider how to feed the result of the operation plan optimization problem $\left(\mathrm{P}_k(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{z}})\right)$ back into the equipment configuration $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$. To simplify the notation, $\left(\mathrm{P}_k(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{z}})\right)$ is expressed as follows:

$$\left(\mathrm{P}_k(\bar{\boldsymbol{w}})\right): \underset{\boldsymbol{u}_k, \boldsymbol{v}_k}{\text{minimize}} \; \boldsymbol{c}_k^{\top}\boldsymbol{u}_k \quad \cdots\cdots(36)$$

subject to

$$\boldsymbol{A}_k\boldsymbol{u}_k + \boldsymbol{B}_k\boldsymbol{v}_k \ge \boldsymbol{d}_k(\bar{\boldsymbol{w}}), \quad \cdots\cdots(37)$$

$$\boldsymbol{u}_k \ge \boldsymbol{0}, \quad \cdots\cdots(38)$$

$$v_{ki} \in \{0, 1\} \quad \left(i \in 1, .., N_{v_k}\right), \quad \cdots\cdots(39)$$

where the symbol $\bar{\boldsymbol{w}}$ denotes the equipment configuration $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{z}})$, $\boldsymbol{u}_k$ denotes an N_{u_k}-dimensional non-negative continuous variable vector, v_k denotes an N_{v_k}-dimensional 0-1 variable vector, and $\boldsymbol{A}_k$, $\boldsymbol{B}_k$, $\boldsymbol{c}_k$, and $\boldsymbol{d}_k(\bar{\boldsymbol{w}})$ denote matrices and vectors of appropriate sizes. Note that, other than $\boldsymbol{d}_k(\bar{\boldsymbol{w}})$, these values do not depend on the equipment configuration $\bar{\boldsymbol{w}}$.

The following is the continuous relaxation problem for the operation plan optimization problem $\left(\mathrm{P}_k(\bar{\boldsymbol{w}})\right)$:

$$\left(\tilde{\mathrm{P}}_k(\bar{\boldsymbol{w}})\right): \underset{\boldsymbol{u}_k, \boldsymbol{v}_k}{\text{minimize}} \; \boldsymbol{c}_k^{\top}\boldsymbol{u}_k \quad \cdots\cdots(40)$$

subject to

$$\boldsymbol{A}_k \boldsymbol{u}_k + \boldsymbol{B}_k \boldsymbol{v}_k \geq \boldsymbol{d}_k(\bar{\boldsymbol{w}}), \quad \cdots (41)$$

$$\boldsymbol{u}_k \geq \boldsymbol{0}, \quad \cdots (42)$$

$$\boldsymbol{0} \leq \boldsymbol{v}_k \leq \boldsymbol{1}. \quad \cdots (43)$$

And its dual problem is as follows:

$$\left(\widetilde{\mathrm{D}}_k(\bar{\boldsymbol{w}})\right): \underset{\boldsymbol{\lambda}_k, \boldsymbol{\mu}_k}{\text{maximize}}\ \boldsymbol{d}_k^\top(\bar{\boldsymbol{w}})\boldsymbol{\lambda}_k - \boldsymbol{1}^\top \boldsymbol{\mu}_k \quad \cdots (44)$$

subject to

$$\boldsymbol{A}_k^\top \boldsymbol{\lambda}_k \leq \boldsymbol{c}_k, \quad \cdots (45)$$

$$\boldsymbol{B}_k^\top \boldsymbol{\lambda}_k - \boldsymbol{1}^\top \boldsymbol{\mu}_k \leq \boldsymbol{0}, \quad \cdots (46)$$

$$\boldsymbol{\lambda}_k \geq \boldsymbol{0}, \quad \cdots (47)$$

$$\boldsymbol{\mu}_k \geq \boldsymbol{0}. \quad \cdots (48)$$

Since, by assumption, $\left(\mathrm{P}_k(\bar{\boldsymbol{w}})\right)$ has an optimal solution, its relaxation $\left(\widetilde{\mathrm{P}}_k(\bar{\boldsymbol{w}})\right)$ also has an optimal solution obviously and, according to the duality of linear programming problem, $\left(\widetilde{\mathrm{D}}_k(\bar{\boldsymbol{w}})\right)$ also has an optimal solution. Let the optimal values of $\left(\mathrm{P}_k(\bar{\boldsymbol{w}})\right)$, $\left(\widetilde{\mathrm{P}}_k(\bar{\boldsymbol{w}})\right)$, and $\left(\widetilde{\mathrm{D}}_k(\bar{\boldsymbol{w}})\right)$ denote as $f^*\left(\mathrm{P}_k(\bar{\boldsymbol{w}})\right)$, $f^*\left(\widetilde{\mathrm{P}}_k(\bar{\boldsymbol{w}})\right)$, and $f^*\left(\widetilde{\mathrm{D}}_k(\bar{\boldsymbol{w}})\right)$, respectively. Note that the following relation holds:

$$f^*\left(\widetilde{\mathrm{D}}_k(\bar{\boldsymbol{w}})\right) = f^*\left(\widetilde{\mathrm{P}}_k(\bar{\boldsymbol{w}})\right) \leq f^*\left(\mathrm{P}_k(\bar{\boldsymbol{w}})\right) \quad \cdots (49)$$

The feasible solution set of the continuous relaxation dual problem $\left(\widetilde{\mathrm{D}}_k(\bar{\boldsymbol{w}})\right)$ is as follows:

$$\mathcal{D}_k = \left\{ (\boldsymbol{\lambda}_k, \boldsymbol{\mu}_k) \;\middle|\; \begin{array}{c} \boldsymbol{A}_k^\top \boldsymbol{\lambda}_k \leq \boldsymbol{c}_k, \\ \boldsymbol{B}_k^\top \boldsymbol{\lambda}_k - \boldsymbol{\mu}_k \leq \boldsymbol{0}, \\ \boldsymbol{\lambda}_k \geq \boldsymbol{0}, \\ \boldsymbol{\mu}_k \geq \boldsymbol{0} \end{array} \right\} \quad \cdots (50)$$

Since it does not depend on the equipment configuration $\bar{\boldsymbol{w}}$, the following relation holds for any $\bar{\boldsymbol{w}}$ and $(\boldsymbol{\lambda}_k, \boldsymbol{\mu}_k) \in \mathcal{D}_k$:

$$\boldsymbol{d}_k^\top(\bar{\boldsymbol{w}})\boldsymbol{\lambda}_k - \boldsymbol{1}^\top \boldsymbol{\mu}_k \leq f^*\left(\widetilde{\mathrm{D}}_k(\bar{\boldsymbol{w}})\right) \leq f^*\left(\mathrm{P}_k(\bar{\boldsymbol{w}})\right) \quad \cdots (51)$$

This means that, when determining $\bar{\boldsymbol{w}}$, Expression (51) can be used as a cutting plane to tighten the lower bound of f_k^{O}. In particular, the following equation holds for $\left(\lambda_k^*(\bar{\boldsymbol{w}}), \mu_k^*(\bar{\boldsymbol{w}})\right)$, which is the optimal solution of $\left(\widetilde{\mathrm{D}}_k(\bar{\boldsymbol{w}})\right)$.

$$\boldsymbol{d}_k^\top(\bar{\boldsymbol{w}})\boldsymbol{\lambda}_k^*(\bar{\boldsymbol{w}}) - \boldsymbol{1}^\top \boldsymbol{\mu}_k^*(\bar{\boldsymbol{w}}) = f^*\left(\widetilde{\mathrm{D}}_k(\bar{\boldsymbol{w}})\right) \quad \cdots (52)$$

Therefore, the cutting plane based on the optimal solution $\left(\lambda_k^*(\bar{\boldsymbol{w}}), \mu_k^*(\bar{\boldsymbol{w}})\right)$ gives the strongest lower bound. The algorithm proposed in this paper determines equipment configuration $\bar{\boldsymbol{w}}$ and then, solves $\left(\widetilde{\mathrm{D}}_k(\bar{\boldsymbol{w}})\right)$ in order to obtain the cutting plane and tighten the lower bound. This process is repeatedly performed so as to obtain a good approximate solution of the original problem (P). **Figure 4** gives a summary of the proposed algorithm. **Figure 5** shows the corresponding flowchart.

5. Numerical experiment

5.1 Conditions

In this section, a numerical experiment is conducted on the proposed model and algorithm, taking it as an example to discuss introduction of a lithium-ion battery and a gas engine generator aimed at reducing the electricity cost at a virtual factory.

This numerical experiment uses an energy system configuration and operation optimization system that was developed based on the model described in **Chapter 3** and the algorithm described in **Chapter 4**. **Figure 6** shows example screenshots of the system. Using this system makes it possible to perform a series of investigations, without having to be aware of the mathematical model, such as adding resources and equipment that are being considered, setting parameters, executing optimization calculations, and visualizing results. All of the numerical experiment conditions mentioned below can be specified on the screens of this system.

Figure 7 shows the pattern of expected electricity demand. The demand is high during daytime working hours (8:30 to 17:30), and temporarily low during lunch break (12:00 to 13:00). One means of meeting electricity demand is to purchase electricity from the commercial grid. If the unit price of purchased electricity is different for daytime and nighttime, then electricity purchase cost could be reduced by charging a battery when the unit price is low and by discharging it when the unit price is high. In addition, if the cost per kW·h of generating electricity with a gas engine generator is lower than the unit price of purchased electricity, then it may be advantageous to introduce it. However, introducing a battery and a gas engine generator requires an initial investment, and profitability must be considered based on their durable life. Other calculation conditions are given in **Table 6**. The information on cost and efficiency is set based on References (10) to (12). The unit price of purchased electricity, which differs depending on time of day, is taken to be as follows:

- 12.77 yen/kW·h (nighttime: 0:00 to 8:00, 22:00 to 24:00)
- 18.54 yen/kW·h (daytime: 8:00 to 13:00, 16:00 to 22:00)
- 19.20 yen/kW·h (peak: 13:00 to 16:00)

Table 7 shows the calculation environment used for the optimization calculations.

5.2 Results

Table 8 shows the equipment configuration obtained under the conditions described in **Section 5.1**. **Figure 8** shows the operation plan for each piece of equipment for electricity demand in the final year, as obtained using the proposed method.

5.3 Discussion

5.3.1 Equipment configuration and operation pattern

As shown in **Table 8**, as a result of optimization, a solution was obtained in which only a gas engine is introduced. The reason for not introducing a battery is that (under these conditions), although a reduction in electricity purchase cost is achieved by charging and discharging a battery in consideration of the difference in purchased electricity unit price according to time of day, this reduction is smaller than the initial cost of introducing the battery.

We now discuss the gas engine operation pattern obtained as a result of optimization. **Figure 8** shows the pattern obtained when a gas engine is operated only between 8:00 and 22:00. This corresponds to daytime and peak hours, when the unit price of purchased electricity is high. From **Table 6**, it can be calculated that the unit cost of generating electricity with a gas engine is 15.49 yen/kW·h. The unit

Step 0 : (Initialization)

Set the number of iterations t to 0, upper bound U_{-1} to $+\infty$, lower bound L_{-1} to $-\infty$, and dual optimal solution set as $\mathcal{D}'_{tk} = \phi \, (k \in \mathcal{K})$.

Step 1 : (Determination of equipment configuration)

Solve the following optimization problem:

$$(\mathrm{MP}_t): \underset{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\xi}{\text{minimize}} \; f^{\mathrm{I}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) + \sum_{k \in \mathcal{K}} f_k^{\mathrm{M}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) + \xi \quad \cdots\cdots (53)$$

subject to

$$x_i^{\min} z_i \le x_i \le x_i^{\max} z_i \quad (i \in \mathcal{E}), \quad \cdots\cdots (54)$$

$$y_i^{\min} z_i \le y_i \le y_i^{\max} z_i \quad (i \in \mathcal{S}), \quad \cdots\cdots (55)$$

$$\sum_{k \in \mathcal{K}} \left(\boldsymbol{d}_k^{\top}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) \boldsymbol{\lambda}_k - \mathbf{1}^{\top} \boldsymbol{\mu}_k \right) \le \xi \quad \left((\boldsymbol{\lambda}_k, \boldsymbol{\mu}_k) \in \mathcal{D}'_{tk} \right), \quad \cdots\cdots (56)$$

$$z_i \in \{0,1\} \quad (i \in \mathcal{E}), \quad \cdots\cdots (57)$$

$$\xi > -\infty \; . \quad \cdots\cdots (58)$$

Denote the obtained optimal value as $f^*(\mathrm{MP}_t)$, and optimal solution as $(\boldsymbol{x}_t^*, \boldsymbol{y}_t^*, \boldsymbol{z}_t^*, \xi_t^*)$. The symbol $\boldsymbol{d}_k$ in Expression (56) is as in the form of the operation plan problem $(\mathrm{P}_k(\bar{\boldsymbol{w}}))$, and is a linear expression of $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$.

Step 2 : (Update of lower bound value)

Based on $f^*(\mathrm{MP}_t)$ obtained in Step 1, update the lower bound value of the original problem (P) as follows:

$$L_t = \max\left(L_{t-1}, f^*(\mathrm{MP}_t)\right) \quad \cdots\cdots (59)$$

Step 3 : (Optimization of operation plan)

Based on the optimal solution $(\boldsymbol{x}_t^*, \boldsymbol{y}_t^*, \boldsymbol{z}_t^*, \xi_t^*)$ obtained in Step 1, denote the equipment configuration as $\bar{\boldsymbol{w}}_t = (\boldsymbol{x}_t^*, \boldsymbol{y}_t^*, \boldsymbol{z}_t^*)$ solve the operation plan optimization problem $(\mathrm{P}_k(\bar{\boldsymbol{w}}_t))$ for $k \in \mathcal{K}$, and denote the optimal solution as $\left(\boldsymbol{p}_k^*(\bar{\boldsymbol{w}}_t), \boldsymbol{p}_k^{+*}(\bar{\boldsymbol{w}}_t), \boldsymbol{p}_k^{-*}(\bar{\boldsymbol{w}}_t), \boldsymbol{q}_{k0}^*(\bar{\boldsymbol{w}}_t), \boldsymbol{h}_k^*(\bar{\boldsymbol{w}}_t), \boldsymbol{s}_k^{+*}(\bar{\boldsymbol{w}}_t), \boldsymbol{s}_k^{-*}(\bar{\boldsymbol{w}}_t) \right) (k \in \mathcal{K})$.

Step 4 : (Update of upper bound value)

Solve the equation below, based on the equipment configuration $\bar{\boldsymbol{w}}_t = (\boldsymbol{x}_t^*, \boldsymbol{y}_t^*, \boldsymbol{z}_t^*)$ obtained in Step 1, and $\left(\boldsymbol{s}_k^{+*}(\bar{\boldsymbol{w}}_t), \boldsymbol{s}_k^{-*}(\bar{\boldsymbol{w}}_t) \right)$ which is part of the optimal solution of the operation plan problem for each year obtained for $\bar{\boldsymbol{w}}_t$ in Step 3:

$$f_t \left(\boldsymbol{x}_t^*, \boldsymbol{y}_t^*, \boldsymbol{z}_t^*, \boldsymbol{s}_k^{+*}(\bar{\boldsymbol{w}}_t), \boldsymbol{s}_k^{-*}(\bar{\boldsymbol{w}}_t) \right) = f^{\mathrm{I}}(\boldsymbol{x}_t^*, \boldsymbol{y}_t^*, \boldsymbol{z}_t^*) + \sum_{k \in \mathcal{K}} f_k^{\mathrm{R}} \left(\boldsymbol{x}_t^*, \boldsymbol{y}_t^*, \boldsymbol{z}_t^*, \boldsymbol{s}_k^{+*}(\bar{\boldsymbol{w}}_t), \boldsymbol{s}_k^{-*}(\bar{\boldsymbol{w}}_t) \right) \quad \cdots\cdots (60)$$

Then, update the upper bound value of the original problem (P) as follows:

$$U_t = \min (U_{t-1}, f_t) \quad \cdots\cdots (61)$$

Step 5 : (Optimization of continuous relaxation dual problem for operation plan)

For $k \in \mathcal{K}$, solve the continuous relaxation dual problem $(\tilde{\mathrm{D}}_k(\bar{\boldsymbol{w}}_t))$ for the operation plan optimization problem $(\mathrm{P}_k(\bar{\boldsymbol{w}}_t))$. Denote the optimal solution as $\left(\boldsymbol{\lambda}_k^*(\bar{\boldsymbol{w}}_t), \boldsymbol{\mu}_k^*(\bar{\boldsymbol{w}}_t) \right) (k \in \mathcal{K})$.

Step 6 : (Update of dual optimal solution set)

Based on $\left(\boldsymbol{\lambda}_k^*(\bar{\boldsymbol{w}}_t), \boldsymbol{\mu}_k^*(\bar{\boldsymbol{w}}_t) \right) (k \in \mathcal{K})$ obtained in Step 5, update the dual optimal solution set $\mathcal{D}'_{tk}$ as follows:

$$\mathcal{D}'_{t+1k} = \mathcal{D}'_{tk} \cup \left(\boldsymbol{\lambda}_k^*(\bar{\boldsymbol{w}}_t), \boldsymbol{\mu}_k^*(\bar{\boldsymbol{w}}_t) \right) \quad (k \in \mathcal{K}) \quad \cdots\cdots (62)$$

Step 7 : (Convergence check)

Specify an appropriate tolerance. If either of the following conditions is satisfied, then output the decision variable for the upper bound value as an incumbent solution, and terminate calculation.

(a) $U_t = L_t$

(b) $U_t = U_{t-1}$ and $L_t = L_{t-1}$

If neither is satisfied, then set $t := t + 1$, and go back to Step 1.

Fig. 4 Optimization Algorithm

price of purchased electricity between 8:00 and 22:00 is 18.54 yen or more, and that between 22:00 and 8:00 of the following day is 12.77 yen. This shows that the gas engine operation pattern obtained as a result of optimization is rational, with the gas engine being operated only during those hours when the unit cost of gas engine electricity generation is lower than the unit price of purchased electricity.

Next, we discuss the output of the gas engine (6 000 kW) obtained as a result of optimization. From the discussion above, there are hours during which the unit cost of gas engine electricity generation is lower than the unit price of purchased electricity, and therefore, in order to maximize economic advantage, it is reasonable to introduce a 6 000 kW gas engine, which is at the upper limit of the output range. In such a case, there is a concern that the total cost will change if the output range of the gas engine is increased further. With regard to this, information useful for sensitivity analysis can be obtained by defining and solving an appropriate dual problem. **Table 9** shows the cost improvement sensitivity obtained when the output and capacity ranges of each piece of equipment are increased by 1 kW (1 kW·h). The table

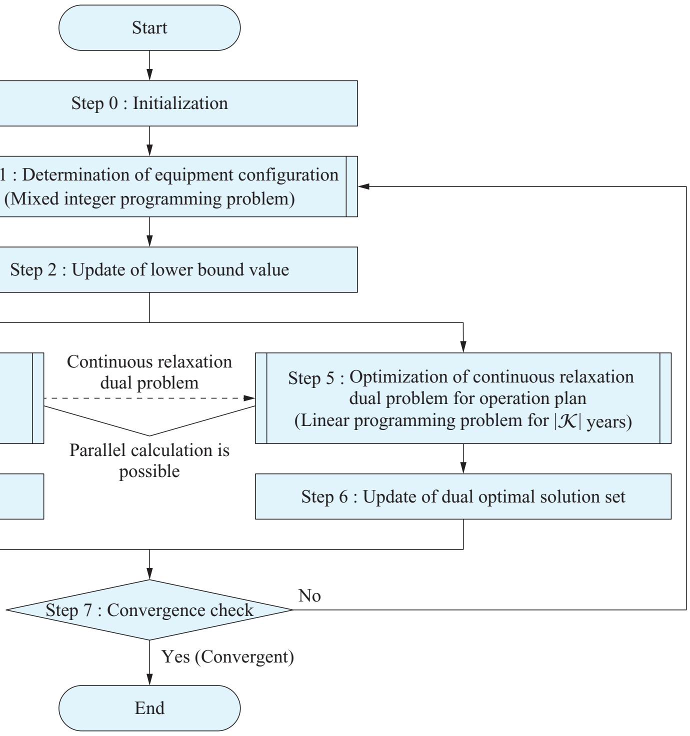


Fig. 5 Flow-chart of the proposed algorithm

(a) Variation in cost incurred in each year and breakdown

(b) Optimal operation plan for typical day

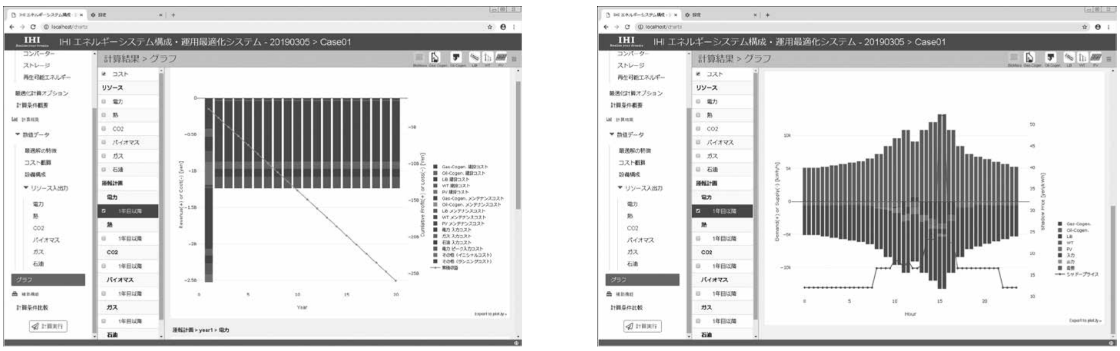


Fig. 6 Screenshots of the optimization system for energy system configuration and operation

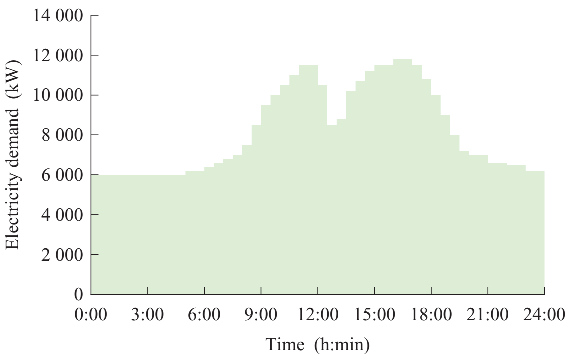


Fig. 7 Electricity demand profile

shows the values of the dual optimal solutions for Expressions (3) and (4) of a linear programming problem obtained by fixing the 0-1 variables in the original problem (P) at their incumbent solution values. **Table 9** shows that by increasing the upper limit of the gas engine's output range by 1 kW the total cost can be reduced by 70 000 yen.

5.3.2 Calculation time of optimization algorithm

The calculation took 27 seconds, which is reasonable with respect to the requirement given in **Section 3.1**. The problem settings for this example calculation are relatively simple, and it was confirmed that the optimal solution can be obtained with similar calculation time even when using simple application of a mathematical programming solver. The difference from the proposed method becomes more significant with larger and more complicated problems. When using simple application of a mathematical programming solver, a calculation time of two or more days may be required if the number of pieces of equipment or number of resource types is increased. However, even in such cases, it has been empirically confirmed that a reasonable solution can be obtained in approximately one minute using the proposed method.

Table 6 Calculation Condition

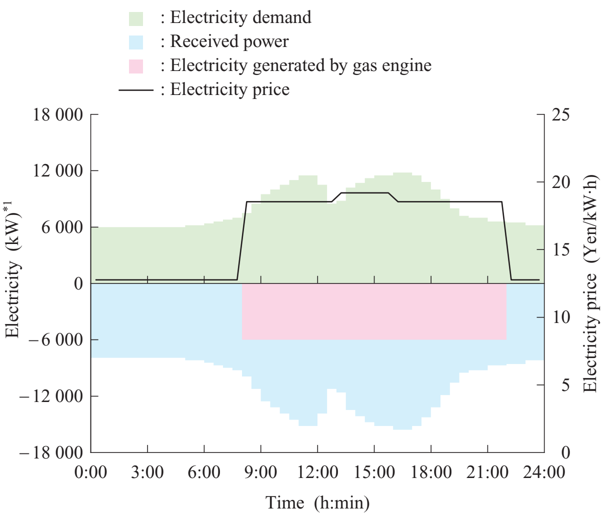
Item		Unit	Value
Battery	Output range	kW	500 – 3 000
	Capacity range	kW·h	500 – 3 000
	Efficiency	%	95 (one side)
	Initial cost[10]	10 000 yen/kW·h	15 9
	Maintenance cost	10 000 yen/kW·h·y 1 000 yen/kW·y	1.5 9
	Operating SOC range	%	10 – 90
Gas engine	Output range	kW	3 000 – 6 000
	Generation efficiency[11]	%	44.0
	Initial cost[11]	10 000 yen/kW	1.21
	Maintenance cost[11]	10 000 yen/y	1.0
	Minimum output	%	100 (rated operation only)
Electricity	Unit price of purchased electricity[12]	Yen/kW·h	12.77 – 19.20 (depends on time of day)
	Electricity basic charge[12]	Yen/kW·month	1 815
	Demand growth rate	%/y	2
Gas	Unit purchase price[11]	Yen/MJ	1.85
Number of years during which cost is considered		y	15

Table 7 Calculation Environment

Item	Value
OS	Windows 7 Enterprise 64bit SP1
CPU	Intel® Xeon® CPU E-1505M v5
Memory	64 GB
Mathematical programming solver	Cbc 2.9.0[13]

Table 8 Optimization Result

Item		Unit	Value
Battery	Output	kW	0
	Capacity	kW·h	0
Gas engine	Output	kW	6 000
Calculation time		s	27
Total cost		10 000 yen	2 369 098



(Note) *1 : Demand side is positive and supply side is negative

Fig. 8 Optimal operation plan for the electricity demand of the final year

Table 9 Cost improvement sensitivity when the upper and lower limits of output and capacity of each facility are enlarged by 1 kW (1 kW·h)

Item		Unit	Lower limit side	Upper limit side
Battery	Output	Yen/kW	0	0
	Capacity	Yen/kW·h	0	0
Gas engine	Output	10 000 yen/kW	0	7

6. Conclusion

In this paper, we proposed a versatile mathematical model for optimizing the configuration and operation of energy systems and an efficient approximate algorithm which exploits the mathematical structure of the model, and they have been verified through a numerical experiment. The proposed model and algorithm have been implemented as an optimization system that eliminates the need to develop a model or algorithm for each project and is expected to improve efficiency when studying the optimization of energy system configuration and operation. In addition, even when new renewable energies and the energy equipment that uses them emerge, it will be possible to make immediate proposals regarding how these can be effectively combined.

One future challenge is, to develop an operation optimization model and algorithm that can be widely applied to projects in their more advanced phases, such as examining detailed operation methods for each piece of energy equipment after equipment configuration has been determined. The model proposed in this paper is intended for use mainly as an aid when studying equipment configuration, while also taking equipment operation into consideration; it does not take the detailed characteristics of each piece of energy equipment into consideration, such as changes in efficiency with varying output, or responsiveness. When developing a detailed operation optimization model, it is necessary to discuss not only these hardware aspects but also intangible ones, such as equipment operation rules. Particularly with respect to intangible aspects, different factories and plants have different approaches, therefore complete generalization is difficult. We consider that software design needs to be improved such that, while providing a model that covers some typical rules, it is also possible to add constraints for each project.

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