

Study of Shape Design via Principal Component Analysis and its Optimization

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Generally, the design of a shape is expressed by a combination of design parameters such as length and angle, and the optimal shape is investigated according to some optimization method. By using a general parameterization method, the expressibility of the optimal design is constrained by how design parameters are selected, possibly making the optimal design too difficult to find. In this paper, a new shape parameterization method using principal component analysis is proposed so as to express the shape with a greater degree of freedom. Also, an example of shape optimization for film cooling holes in turbine blades using the proposed parameterization method and TDM is introduced.

1. Introduction

Thanks to recent advances in computer technology, shape optimization using Computer-Aided Engineering (CAE) such as the Finite Element Method (FEM) and Computational Fluid Dynamics (CFD) is an active area of design. Conducting shape optimization with CAE requires a lot of numerical analyses. Investigating optimizations more quickly and over a wider range requires efficiency improvements achieved by reducing labor and automating the analysis technology.⁽¹⁾ However, expressing a complex shape requires many design parameters, and since the required computation correspondingly becomes more complex, there is a limit to the benefits provided by efficiency improvements alone. For this reason, different innovations for reducing the computational complexity are becoming necessary in order to stay within realistic computation times. One method for solving this issue involves combining experimental design methods with response surface methodology, evaluating the performance of each shape in an approximate manner with a small number of analysis iterations, and obtaining an optimal shape. Obviously, as computer technology continues to develop, the combination of experimental design and response surface methodology may be replaced with direct CAE analysis of each shape.

Another potential technique for reducing computational complexity is reducing the number of design parameters. Even if a large number of design parameters exist, the number of parameters with greater sensitivity to the performance to be evaluated is limited. Thus, parameters that do not affect performance may be held constant to reduce the number of parameters. A mathematical technique called

dimension reduction can also be used to reduce the number of design parameters. The present study uses this technique for shape creation.

There is a variety of widely used methods of defining parameters to vary and optimize shapes. The authors⁽²⁾ have proposed a methodology called Total Design Management (TDM), in which solutions are first found over the whole range of design parameters, and from these, solutions that satisfy certain conditions are selected. With parameterized shape optimization, parameters are arbitrarily varied within a design range, and by evaluating the performance of the resulting shapes, the combination of parameters yielding optimal performance is found. With more complex shapes, the number of independent parameters forming the shape increases, and as the number of parameters increases, the number of candidate shapes to evaluate becomes extremely large. For this reason, fewer parameters are preferable. Additionally, there is a limited number of shapes that can be produced based on parameters such as lengths and angles, so an optimal shape may not necessarily be able to be found.

The shape representation proposed in this study is a new technique using principal component analysis. Several representative shapes are collected in advance, and by applying principal component analysis to those shapes, characteristic parameters contained in those shapes are found. These parameters are then set as the design parameters, and new shapes are generated by taking a linear sum of the basis vectors. At this point, the parameters obtained by principal component analysis can be used to create most shapes with several lower-order parameters. As a result, by ignoring the higher-order parameters, the number of design parameters can be reduced. Also, by setting new parameters not based on lengths, angles or other such

properties, it is possible to generate many kinds of shapes, and expand the possibility of finding an optimal shape.

This article discusses how shape representation using principal component analysis combined with TDM is effective as a technique for finding an optimal shape.

2. Investigation methods

2.1 Shape representation using principal component analysis

In this study, new parameters obtained by principal component analysis are introduced as design parameters for representing shapes.⁽³⁾ By correlating predefined shapes using principal component analysis, new shape parameters are obtained. Moreover, by only using strongly correlative components, the number of parameters is also reduced.

Although there are several methods for carrying out principal component analysis, snapshot Proper Orthogonal Decomposition (POD)⁽⁴⁾ is used here to carry out calculations. **Figure 1** illustrates an overview of shape representation by principal component analysis. First, n shapes $\{\phi_i (i = 1, \dots, n)\}$ prepared in advance are provided. Here, ϕ_i is the set of discrete point sequences that construct the i th shape. The average of ϕ_i is expressed as follows.

$$ave(\phi_i) = \frac{1}{n} \sum_{i=1}^n \phi_i \dots\dots\dots (1)$$

Also, the quantity obtained by subtracting the average from each ϕ_i is defined.

$$\hat{\phi}_i = \phi_i - ave(\phi_i) \dots\dots\dots (2)$$

Next, a tensor P is defined as the inner product of ϕ_i .

$$P_{ij} = \langle \hat{\phi}_i, \hat{\phi}_j \rangle \dots\dots\dots (3)$$

The eigenvalue λ_l and eigenvector v_l of P are then calculated as follows.

$$Pv_l = \lambda_l v_l \dots\dots\dots (4)$$

Here, the eigenvalues are assumed to be arranged in descending order. The i th-order principal component ψ_i is

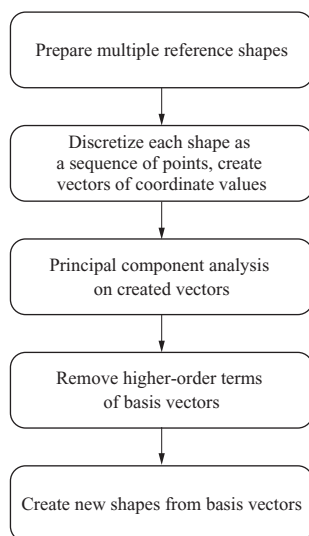


Fig. 1 Flow chart of shape design by principal component analysis

defined as follows.

$$\psi_i = \sum_j v_{ij} \hat{\phi}_j \dots\dots\dots (5)$$

Here, v_{ij} indicates the i th-order element of the vector v_j .

Additionally, i is arranged so that the norm on $\|\psi_i\|$ is in descending order. By using ψ_i as basis vectors, the shape Φ can be expressed as follows.

$$\Phi = \left\{ \phi \mid \phi = \sum_i^{n-1} \alpha_i \psi_i + ave(\phi_i), \alpha_i \in \mathfrak{R} \right\} \dots\dots\dots (6)$$

Here, α_i are parameters. By multiplying the basis vectors by specific parameters, the base shape can be completely reproduced. Also, by freely setting parameters, the base shape can be modified to generate other shapes.

Since $\|\psi_1\| \geq \|\psi_2\| \geq \dots \geq \|\psi_n\|$, ψ_j exerts less influence on the shape than ψ_i (where $i < j$). In order to reduce the number of parameters by utilizing this property, higher-order terms that exert less influence on the shape are omitted to generate the shape Φ^{red} .

$$\Phi^{red} = \left\{ \phi \mid \phi = \sum_{j=1}^N \alpha_j \psi_j + ave(\phi_j), \alpha_j \in \mathfrak{R} \right\} \dots\dots\dots (7)$$

Here, N is the number of principal components ψ_i from the first-order term to the term before which all higher-order terms are omitted.

To decide on an N , a basis vector contribution ratio d_i and a cumulative contribution ratio D_i are defined.

$$d_i = \frac{\|\psi_i\|}{\sum_{j=1}^n \|\psi_j\|} \dots\dots\dots (8)$$

$$D_i = \sum_{j=1}^{j=i} d_j \dots\dots\dots (9)$$

The number N of principal components to be used is decided using d_i and D_i . Potential selection methods include the following.

- (1) Lining up the contribution ratios d_i in descending order, and setting N so that d_i is equal to or greater than a certain value
- (2) Setting N so that the cumulative contribution ratio D_i is equal to or greater than a certain value
- (3) Defining a fixed N at the beginning of the calculation

In this study, method (2) was selected. Reducing the number of principal components means that the base shape can no longer be fully represented, but the degree of reproducibility can be estimated using D_i . When carrying out optimization, representing the base shape is not very meaningful, and so low reproducibility does not pose a problem. Evaluating and finding more shapes is more important.

2.2 Optimization using TDM

This study adopts the set-based design methodology of TDM (**Fig. 2**).⁽²⁾ With set-based design, first, a design object is converted into a mathematical model, and the total set of design solutions is created from the model. Next, solutions

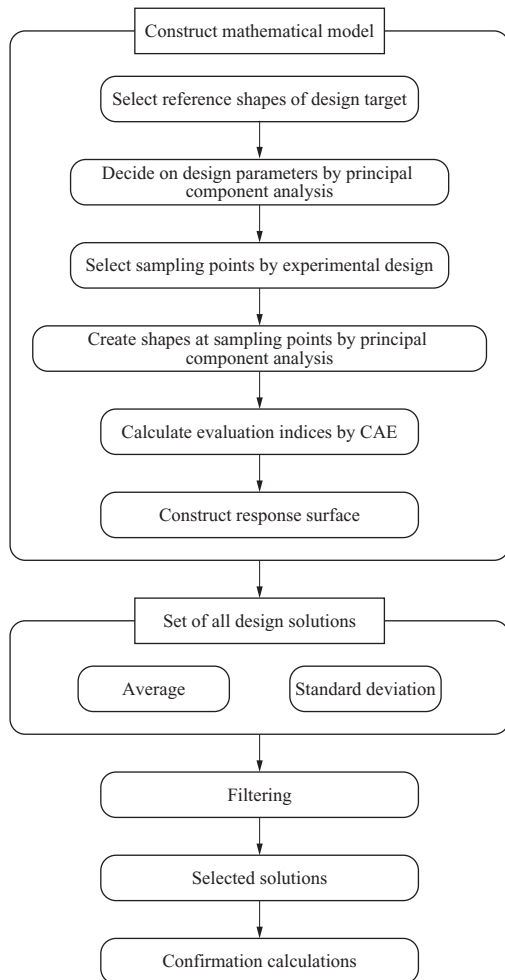


Fig. 2 Flow chart of TDM using set based design

that meet constraints and performance requirements are selected by filtering, and an optimal solution is obtained.

The mathematical model is a group of formulas that transform design parameters into evaluation indices. This study considers the mathematical model to be the transformations of the design parameters obtained by principal component analysis into evaluation indices, and uses the evaluation indices obtained by the application of CAE and response surface methodology. First, design parameters that contribute to shape formation are extracted by principal component analysis from a reference model of the design object. Next, the experimental design method is applied to the extracted design parameters to decide sampling points at which to execute analysis with CAE. After executing analysis with CAE at the sampling points, formulas able to express the evaluation indices over the entire range of the design parameters are created using response surface methodology, and the group of formulas is taken to be the mathematical model.

The complete set of design solutions is created by using response surface methodology to calculate over the entire range of values that the design parameters may take. The complete set of design solutions is then filtered, and the one selected solution extracted via filtering represents

the optimal solution in that set. If there are many selected solutions, the conditions may be narrowed down further. Since the extraction of solutions using response surface methodology includes errors, CAE is finally used to carry out confirmation calculations.

3. Example application of proposed technique

3.1 Overview

As an example application of the proposed technique, optimization was conducted on film cooling holes used to cool turbine blades in gas turbines and jet engines.

A turbine placed immediately after a combustion chamber is subjected to hot combustion gas, and thus the turbine blades must be able to withstand high temperatures. Raising the efficiency of the entire engine requires raising the turbine inlet temperature, and although materials and blade surface coatings able to withstand high temperatures are being actively researched and developed, high thermal resistance of blades cannot be achieved with materials-related technology alone. Consequently, actively cooling the blades is necessary. Film cooling is a typical method of cooling the blade surface. With film cooling, small holes are provided on the blade surface, and cooling air coming from the compressor and bypassing the combustion chamber is made to flow out from the holes and over the blade surface, thereby cooling the blade surface. Cooling performance varies depending on the shape of the holes formed in the blades, and the goal is to develop film cooling holes that provide the greatest cooling performance with the least amount of cooling air.

3.2 Application of technique

In order to generate shapes by using principal component analysis, 18 base shapes were prepared. Figure 3 illustrates some examples of these base shapes. The base shapes that were used include not only shapes with good performance, but also varieties of shapes that may be parameterized.

Principal component analysis was carried out on the 18 base shapes. Figure 4 illustrates the contribution ratios of the POD parameters of the principal components. Using the cumulative contribution ratio illustrated in Fig. 4 as a reference, the first five lower-order parameters were selected as the parameters for representing shape. In this case, the

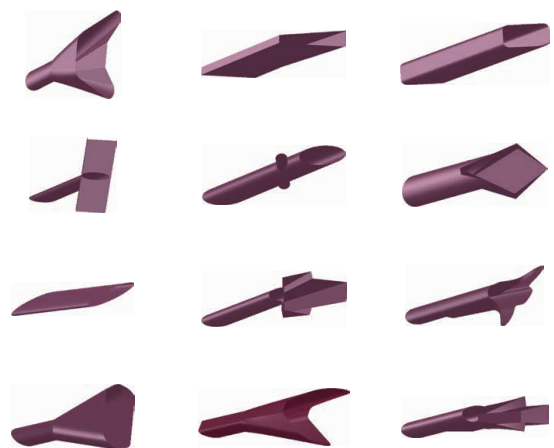


Fig. 3 Examples of reference shapes for cooling hole

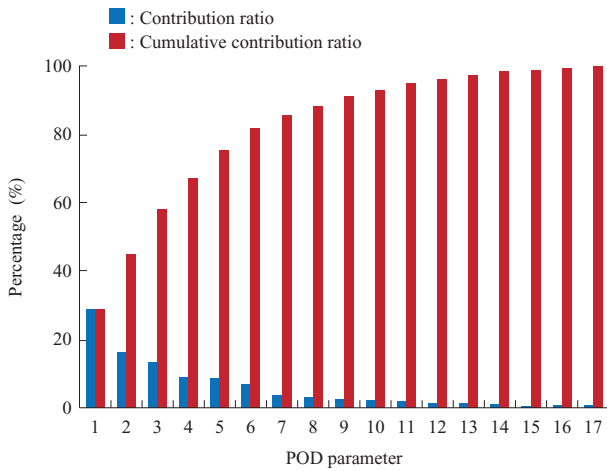


Fig. 4 Contribution ratio of POD parameter

cumulative contribution ratio of these five parameters is approximately 75%. Hereinafter, these parameters are called the POD parameters. Linear sums of the shape variables expressed by these five parameters were taken to define new shapes. **Figure 5** illustrates an example of the newly created shapes.

Next, in order to carry out optimization investigations, indices of fluid design represented by the efficiency of film cooling and indices of structural design represented by the maximum stress produced in a film hole were used as the indices to be evaluated. Results were obtained by using CFD for the indices of fluid design and FEM for the indices of structural design. The CFD analysis (**Fig. 6**) was conducted as a steady flow calculation assuming compressible air, with one film cooling hole provided in the flow passage. This simulates the injection of cooling air into the negative pressure face of the blade. The structural analysis (**Fig. 7**) was conducted by simulating a vibrating test piece and computing the maximum stress. The shapes used in the analysis were created by assigning the five POD parameters to an L36 orthogonal array to obtain combinations of parameters, and then generating shapes from those parameters using principal component analysis. To compute the evaluation indices, a response surface was created on the basis of 36 varieties of CFD and FEM results.

The POD parameters (design parameters) were used at random to generate 10 000 complete design solutions from the response surface, and the solutions were filtered using the fluid design evaluation indices and the structural design



Fig. 5 Examples of creating new shapes

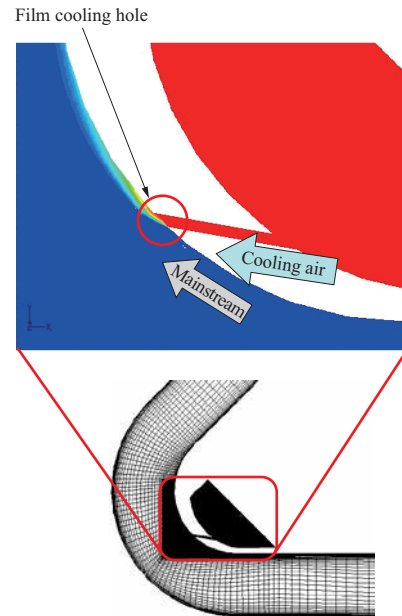


Fig. 6 Computational grid for CFD and example of results in the vicinity of a cooling hole

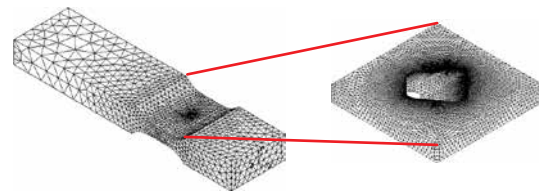


Fig. 7 Computational grid for FEM

evaluation indices (**Fig. 8**). As a result of the filtering, POD parameter combinations primarily indicating two types of shapes were extracted, as illustrated in **Fig. 9**. The POD parameters were normalized from -1 to 1 . These two solutions are hereinafter designated selected solution 1 and selected solution 2. **Figure 10** illustrates complete design solutions and the results selected by filtering. **Figure 10** illustrates the cooling efficiency and the maximum stress, which are the representative evaluation items for all design solutions and the selected solutions. The values in the diagram are normalized with respect to the performance of a film cooling hole with a basic round shape. In **Fig. 10**, solutions closer to the lower-right indicate good performance. Consequently, the selected solutions indicate good performance. The reason why the selected solutions are not at the extreme lower-right in **Fig. 10** is because the solutions were also evaluated according to other evaluation indices not indicated by the axes of the graph. **Figure 11** illustrates the contribution ratio of the five POD parameters with respect to cooling efficiency and maximum stress. In this example application, POD parameters POD3 and POD4 have a large contribution ratio, and there is a possibility that shape optimization can be achieved while further reducing the POD parameters. Additionally, confirmation calculations were carried out for the selected solutions, and **Fig. 12** illustrates the computed distributions of the

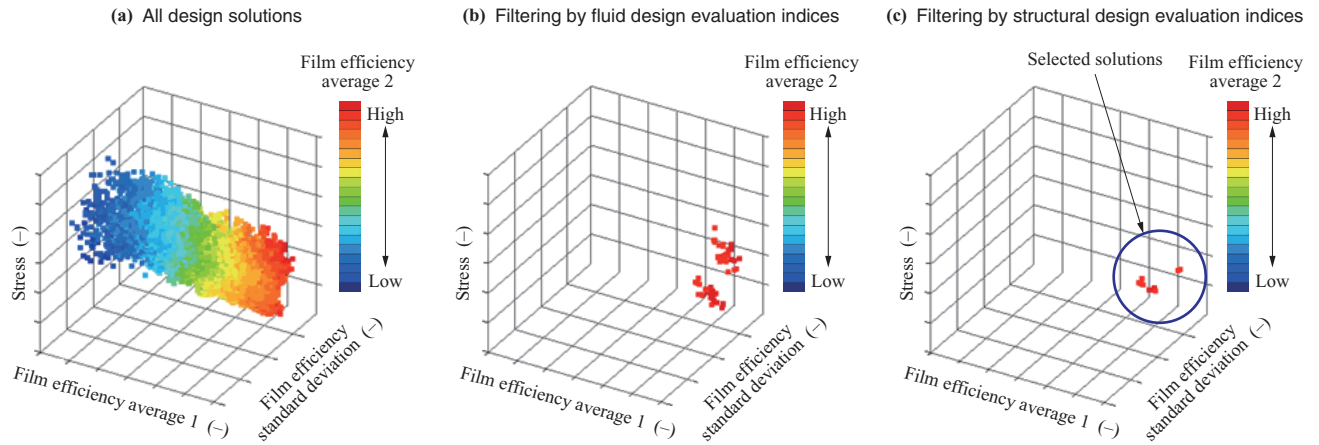


Fig. 8 Filtering process

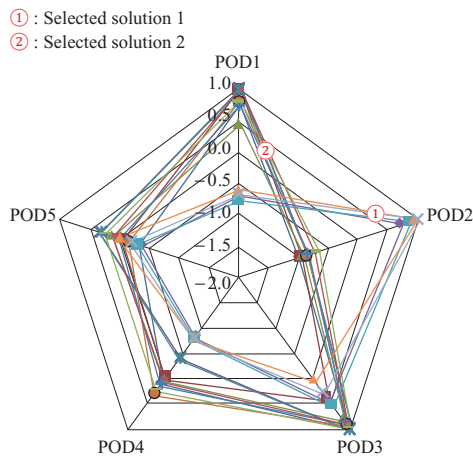


Fig. 9 Result of filtering

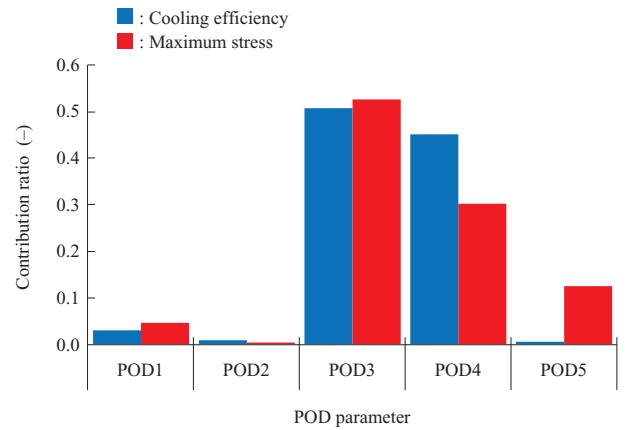


Fig. 11 Contribution ratio of each POD parameter

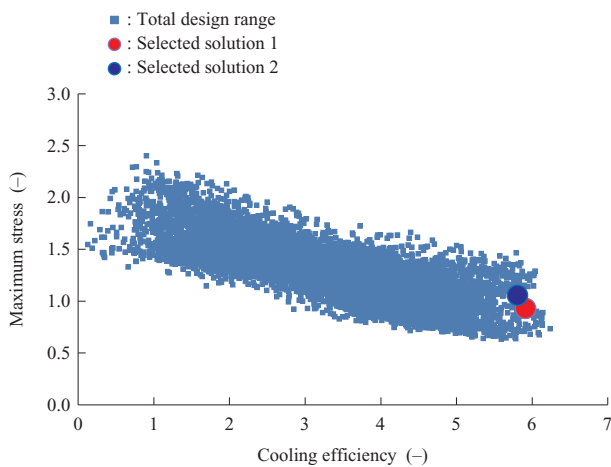


Fig. 10 All design solutions and the selected solution

film cooling efficiency. **Figure 12** demonstrates that the selected solutions are efficient compared to the film cooling efficiency of a round hole.

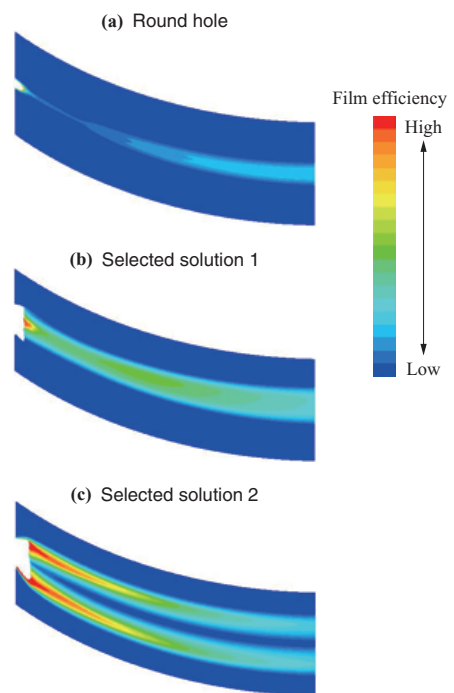


Fig. 12 Distribution of film cooling efficiency

4. Conclusion

Shape representation using principal component analysis was proposed and shown to be applicable to shape optimization. The proposed shape representation method is able to represent a wide variety of shapes with comparatively few design parameters, and is even able to generate shapes that cannot be represented with existing design parameters. By combining this shape representation method with TDM, we believe that many types of optimization investigations can be conducted. As an example application, the optimization of film cooling holes was conducted, and shapes with better performance than a conventional shape were found.

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